4 inches gave perfect signals from Elmer's End, and no disturbance.

Case 3.—On shortening the distant wave-length still more, so as to make it 450 metres, the neighbouring station could not be completely cut out without at the same time introducing a trace of superposed disturbance into the messages received from the distant station.

Case 4.—The difference of wave-length between the two stations was now, therefore, again slightly increased, the Elmer's End wave-length being adjusted to 480 metres, with the local station still remaining at 300.

In this case perfect and strong signals could be received from Elmer's End again, but the separation of the inductive connection had to be as much as 6 inches in order completely to cut out the local signals from the neighbouring station.

It follows, therefore, that when two powerful stations are so excessively near each other as they were in this case—namely, in adjoining fields—a distant signal can be heard with perfect clearness, *i.e.* without any trace of disturbance, only when its wave-length is more than half as great again as that of the neighbouring station; but that undisturbed signalling is much more easy when it approaches double that magnitude, or, of course, when the neighbouring stations are not quite so close together.

In no case was any trace of harmonic detected; *e.g.* when a station was sending 300 metres, and the neighbouring receiving station was attuned to 600 metres, it did not necessarily feel any disturbance. The waves emitted and received by these radiators appear to be practically pure.

OLIVER LODGE.

MARINE BIOLOGY IN THE TORTUGAS.[1]

THE volumes referred to below contain a series of nineteen papers based on work done or material collected at the Marine Biological Laboratory of the Carnegie Institution, situated on Loggerhead Key, off the southwest coast of Florida. The observations recorded bear ample testimony to the exceptionally favourable situation of the laboratory for the prosecution of marine biological research, and also to the facilities afforded on a liberal scale for work on a wide variety of subjects.

Dr. A. G. Mayer, the director of the laboratory, describes the annual breeding swarm of the Atlantic palolo (*Eunice fucata*, Ehlers), which occurs within three days of the day of the last quarter of the moon between June 29 and July 28. The worm when mature (and immature worms take no part in the swarming) is about 10 inches long, and its sexual products are limited to its posterior half. Before sunrise on the day of the annual breeding swarm the worm crawls out backwards from its burrow in the coral or limestone rock until the whole of the sexual portion is protruded. By means of vigorous twisting movements this portion is detached, swims vertically upwards to the surface of the water, and there continues to swim about with its posterior end in front. These sexual portions of the worms, which show no tendency to congregate, are present in great abundance at Tortugas, scarcely a square foot of the surface above the coral reefs being free from them. At sunrise the worms undergo violent contractions, which cause the expulsion of the sexual products through rents or tears which are formed in the body wall; the torn and shrivelled remains of the body wall then sink down to the bottom and die. Although light is probably a contributory cause, it is not the sole cause of this spasm of contraction, which takes place, though it is somewhat delayed, in swimming worms which have been removed to a dark room. After casting off its posterior sexual segments the anterior part of the worm crawls back into its burrow, and regenerates a new sexual end. The author has attempted to determine the nature of the stimulus to which the worm responds when it swarms, and he shows that the worms never swarm when moonlight is prevented from falling upon the rocks in which they are ensconced. The paper is a most interesting contribution to the study of this remarkable phenomenon.

[1] Papers from the Tortugas Laboratory of the Carnegie Institution of Washington. Vol. i., pp. v+191; vol. ii., pp. v+325. (Washington: Carnegie Institution, 1908.)

Dr. Mayer describes a series of experiments on the scyphomedusan *Cassiopea xamachana*, from which he concludes that the stimulus which causes pulsation is due to the constant formation of sodium oxalate in the terminal endoderm cells of the marginal sense organs. The sodium oxalate precipitates calcium as calcium oxalate, thus setting free sodium chloride, which he shows acts as a nervous and muscular stimulant. Pulsation is thus caused by the constant maintenance at the nervous centres in the sense organs of a slight excess of sodium over and above that found in the surrounding sea-water.

The late Prof. W. K. Brooks and Mr. B. McGlone have studied the origin of the lung of Ampullaria. They find that the gills, the lung, and the osphradium arise simultaneously, or nearly so, that they are developed from a ridge or thickening of the mantle, and that they should therefore be regarded as a series of homologous organs specialised among themselves in different directions. The lung becomes functional before the gill, as is shown by the fact that the newly hatched young quickly die if they are prevented from leaving the water, while adults can survive an immersion of a month or more. Other papers, the last productions of the late Prof. Brooks, contain a discussion of the subgenus Cyclosalpa, a description of the rare *Salpa floridana* (Apstein), and of a new appendicularian—*Oikopleura tortugensis*—to the tail of some of which a new species of Gromia was found attached.

Prof. Reighard discusses the significance of the conspicuousness of the coral-reef fishes of the Tortugas. He concludes, as the result of a long series of ingenious experiments, that the coral-reef fishes do not possess that combination of conspicuousness, with unpleasant attributes, necessary to the theory of warning coloration. The conspicuousness of these fishes, since it is not a secondary sexual character and has no necessary meaning for protection, aggression, or as warning, is without biological significance. These fishes have no need of either aggressive inconspicuousness, because they feed chiefly on fixed invertebrates, or of protective inconspicuousness, for they are afforded abundant protection by the reefs and their own agility. Selection has therefore not acted on their colours or other conspicuous characters, but these have developed, unchecked by selection, through internal forces. An attempt is made to apply this conclusion to the "warning coloration" of conspicuous insects.

There are other memoirs on the formation of chromosomes in various echinoderm ova; on the spermatogenesis of the "walking-stick" phasmid, *Aplopus mayeri*, in which the history of the accessory chromosome is traced and its probable significance as a sex determinant discussed; on the habits and reactions of the crab *Ocypoda arenaria*, of Aplopus, and of the woody and sooty terns; on the early development of the scyphozoon Linerges, on actinian larvæ referable to the genera Zoanthella and Zoanthina; on the rate of regeneration in Cassiopea; on regeneration of the chelæ of Portunus, on the life-history of the booby and man-o'-war bird, and on the cestodes of the Tortugas.

THE RELEVANCE OF MATHEMATICS.

ONE of the most important achievements of the thought of the last fifty years has been the conclusive proof of the logical nature of all mathematical conceptions and methods, in opposition to Kant's view that mathematical reasoning is not strictly formal, but always uses *a priori* intuitions of space and time. This does not, of course, imply that the methods of investigation followed by individual mathematicians are essentially different from those followed by other inquirers, the objects of whose researches are not purely logical; it is well known, in fact, that, though a proposition A may logically imply a proposition B, yet B may be deduced from A by considerations quite outside those of logic. Thus the existence of the solution of a certain important and famous mathematical problem—known as "Dirichlet's principle"—was, we may say, *felt*, and actually applied in domains of pure mathematics, for certain physical reasons connected with the equilibrium of statical electricity long before rigorous logical methods were discovered for proving the existence in question. The fact that propositions are

connected logically by no means implies that this connection is obvious, nor does it preclude their being discovered, even in a correct form, by the exercise of what is popularly called "intuition."

By the side of this ever-deepening investigation into the principles of mathematics went on an inquiry, carried on by entirely different men, into the nature and purposes of our conceptions in physics. Through the work of these men, the true relation of mathematics to physical science, which had been a subject on which there had been until then much confusion of thought, appeared clearly. We will glance at the history of mathematics and of the application of mathematics to physics.

From the earliest times until the seventeenth century mathematicians were chiefly occupied with particular questions—the properties of particular numbers and the geometrical properties of particular figures, together with simple mechanical questions concerning centres of gravity, the lever, and so on. The only exception to this was afforded by *algebra*, in which symbols (like our present x and y) took the place of numbers, so that, what is a great advance in economy of thought and other labour,[1] a part of calculation could be done with symbols instead of numbers, so that the *one* result stated a proposition valid for a whole class (often an infinity) of different numbers. Such a result is that which we now write:—

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3,$$

which remains valid when we substitute any particular numbers for a and b, and labour in calculation is often saved by the formula, even in this very simple case.

The great revolution in mathematical thought brought about by Descartes in 1637[2] consists in the application of this general algebra to geometry by the very natural thought of substituting the numbers expressing the lengths of straight lines for those lines. Thus a point in a plane (for instance) is determined in position by two "coordinates" or numbers denoted by x and y, x denoting the distance from a fixed point along a fixed straight line (the abscissa) to a certain point, and y denoting the distance from this last point along a perpendicular (an "ordinate") to the abscissa to the point in question determined by x and y. As the point in question varies in position, x and y both vary; to every x belongs, in general, one or more y's, and we arrive at the most beautiful idea of a single algebraical equation between x and y representing the whole of a curve—the one equation, called the "equation of the curve," expressing the general law by which, given any particular x out of an infinity of them, the corresponding y or y's can be found. Thus $y=3x+2$ gives *one* y for each x, $y^2=3x+2$, or, more generally, $y^2=mx+n$, where m and n stand for any fixed numbers, gives *two* y's, one positive and one negative (above and below the abscissa respectively), for each x, except when x is zero.

The problem of drawing a tangent—the limiting position of a secant, when the two meeting points approach indefinitely close to one another—at any point of a curve came into prominence as a result of Descartes's work, and this, together with the allied conceptions of velocity and acceleration "at an instant"[1] which appeared in Galilei's classical investigation, published in 1638, of the law according to which freely falling bodies move, gave rise at length to the powerful and convenient "infinitesimal calculus of Leibniz and the "calculus of fluxions" of Newton. It is now clearly established that those two methods, which are theoretically—but not practically—the same, were discovered independently; Newton discovered his first, and Leibniz published his first, in 1684. The finding of the areas of curves and of the shapes of the curves which moving particles describe under given forces showed themselves, in this calculus, as results of the inverse process to that of the direct process which serves to find tangents and the law of attraction to a given point from the datum of the path described by a particle. The direct process is called "differentiation," the inverse process "integration."

Newton's fame is chiefly owing to his application of this method to the solution, which, in its broad outlines, he gave, of the problem of the motion of the bodies in the solar system, which includes his discovery of the law according to which all matter gravitates towards (is attracted by) other matter. This was given in his "Principia" of 1687; and, for more than a century afterwards, mathematicians were occupied in extending and applying the calculus.

Of the great mathematicians of this time—the brothers Bernoulli, Euler, Clairaut, d'Alembert, Maclaurin, Lagrange, Laplace, Legendre, Fourier, Poisson, and others —most were Frenchmen; and the successful application of mathematics to celestial and molecular mechanics, to hydrodynamics, to the theory of the conduction of heat, and to electricity and magnetism, brought about, in a great measure, that enthusiastic trust in science, that faith that the whole mystery of life and of our lives was about to be uncovered by it, and that waning of faith in religion, which are so characteristic of France in the eighteenth century, and which are met with in the highest degree in Laplace.

Whether or not it was due to the indirect influence of Kant, whose "Critique of the Pure Reason" first appeared in 1781, an increasing tendency towards critical examination into the validity and the limits of validity of mathematical conceptions and methods appeared in the mathematics of the nineteenth century. First of all we must mention Gauss, who, in an unexampled degree, combined the power of discovery and profound critical insight, so that in the seven volumes of his publications, in the collected edition of his works, there is hardly a page which is not both important in the history of mathematics and free from error. But perhaps of still greater influence was the work of the French mathematician Cauchy; it is he who must be regarded as the chief inspirer—perhaps indirect—of Weierstrass; it is Weierstrass who was the chief inspirer of Georg Cantor, and it is to the influence of Cantor and Dedekind, most of all, that we owe that trend of thought which, with modern mathematical logicians, has resulted in the great discovery of the logical nature of mathematics.

Of course, in this short description there is no implication that the nineteenth century has been poor in the more technical achievements or physical applications of mathematics; in England alone the names of Stokes, Thomson

1 In *The New Quarterly* for October, 1908 (vol. i., p. 498), Mr. N. R. Campbell has objected to the idea of Mach that "economy of thought" is the end for which scientific theories are formed, for reasons based, it seems to me, on a misunderstanding of what Mach really meant. Perhaps the phrase "economy of thought" is not well chosen, and may lead to such misunderstandings; for the principle directs attention to a rule of scientific method which can be readily admitted, and certainly the goal of science, as guided by this principle, will not "have been attained when its students have ceased to think." This rule may be thus described. As science advances, besides actually overcoming an obstacle, it, consciously or unconsciously, leaves marks of guidance for those who come after; so that those obstacles which required great genius to overcome in the first instance afterwards became quite easily so. This is necessary in order that our energies may not be spent by the time that we reach a new obstacle not hitherto surmounted; and "economy of thought" means that we are to be spared waste of the energy of thought whilst treading the path already trodden by our predecessors, so that we may keep it for the really important new problems—not that we may cease to think about problems, new or old.

And thus we have legacies left by great men, such as Lagrange's analytical mechanics and Fourier's theory of the conduction of heat, which are merely inventories of extensive classes of facts, arranged with wonderful compactness. In this description of an *infinity*, perhaps, of facts by a *few* formulæ, there is undoubtedly an æsthetic motive and value; but, apart from this, there is this important economical aspect, that a multitude of particular facts and "laws," which we had hitherto to remember, actually or artificially (in a note-book or library), is, in the theory, comprised in a few symbolical formulæ, which only require *logical* development to get at the particular cases. From this point of view we get the apparent paradox that "economy of thought" leads to the replacing of memory by reason. The solution of the paradox is that logical development can be made more mechanical even than memory, and that thus thought is spared, so that we can concentrate it on the unsolved problems which are always coming into our field of vision as we advance.

The tendency to economy of thought, which is shown in the growth of physics—for example, in the inclusion of the particular Biot's law of the distribution of temperatures in Fourier's theory—may also be seen in the symbolism of pure mathematics.

2 We need hardly point out that this change was not sudden—that Descartes's "Géométrie" was not a "proles sine matre creata," but that here, as everywhere, the development of mathematics has followed the principle of continuity.

1 Mathematically, the finding of the tangent at a point of a curve, and finding the velocity of a particle describing this curve when it gets to that point, are identical problems. They are expressed as finding the "differential coefficient," or the "fluxion" at the point.

(Lord Kelvin), and Maxwell, and those of many living show this; and in Germany one of the greatest influences in pure mathematics was Riemann, who is usually contrasted with Weierstrass as a type of the creative, as opposed to the critical, genius.[1] But in this article we are only concerned with questions in the theory of knowledge, with the principles of mathematics, and the basis of their application to physics, and, through these questions, with the relevance of mathematics to our whole civilisation and, what is still more important, to our whole lives.

The critical inquiries into the nature and purposes of our conceptions in physics, which have been mentioned above, have put in a clear light the fact, which seems to have been overlooked by Laplace in that flush of enthusiasm which a mathematician can so readily understand, and which, without the excuse of the sudden illumination brought about in the eighteenth century by the development of mathematics, is still overlooked by the cruder physicists, that the "world" with which we have to deal in theoretical (mathematical) mechanics, for example, is but a mathematical scheme the function of which it is to imitate by logical consequences of the properties assigned to it by definition certain processes of nature as closely as possible. Thus our "dynamical world" may be called a model of reality, and must not be confused with the reality itself.

That this model of reality is constructed solely out of logical conceptions results from our conclusion that mathematics is based on logic, and on logic alone; that such a model is possible is indeed surprising, and the surprise only goes when we follow up in history the growth of the application of mathematics to physics. The need for completing facts of nature in thought was, no doubt, first felt as a *practical* need—the need that arises because we feel it convenient to be able to predict certain kinds of future events. Thus, with a purely mathematical model of the solar system, we can tell, with an approximation which depends upon the completeness of the model, the relative positions of the sun, stars, and planets several years ahead of time; this enables us to publish the "Nautical Almanac," which is so useful to sailors, and makes up to us, in some degree, for our inability "to grasp this sorry scheme of things entire . . . and re-mould it nearer to the heart's desire."

The need of the completion of facts in thought is not merely practical; it is also intellectual. The striving after logical completeness, whether in generality of results or consistency of its own premisses or those of its models of reality,[3] is accompanied by a feeling of æsthetic pleasure or of intellectual honesty, or of both. We may say that mathematics has an æsthetic and a moral value.

Mathematics is relevant to those who go down to the sea in ships, to those who stay on dry land and build bridges or locomotives, and to those who observe the sun's corona during a total eclipse to find out what the sun is made of. Mathematics is relevant to the philosopher, for not only has it investigated and does it investigate its own foundations, but also it explains what is meant by the philosophers' own phrases, such as "the postulate of the comprehensibility of nature" (which seems to be the postulate that a purely logical model is possible), and the "laws of uniformity, continuity, and causality." And lastly, mathematics, besides being relevant to æsthetics and morals in the above sense, is of moral significance in another respect. Since the basis of mathematics is logic, and logic alone,[1] all those personal, national, and historical questions which are from time to time mixed up with mathematics—however essential some of them may be to the understanding of certain points and to education—show themselves, when looked at from a higher plane of truth, to be irrelevant.

PHILIP E. B. JOURDAIN.

[1] On a closer consideration, this distinction breaks down almost entirely. Apart from the numerous instances which can be quoted of particularly critical work by Riemann and particularly creative work by Weierstrass, surely it is always true both that there should be no creation without criticism (otherwise we run the risk of building castles in the air) and that there cannot be any relevant criticism which does not add to our knowledge, and is in so far creative.

[2] *Cf.* A. Voss, "Über das Wesen der Mathematik." Pp. 3-4. (Leipzig and Berlin: B. G. Teubner, 1908.)

[3] I have tried to show by some examples that we can and ought to examine the details of our models with the aid of the most refined conceptions of modern mathematics, in order to be certain that the models are logically consistent ("On some Points in the Foundation of Mathematical Physics," *The Monist*, April, 1908, vol. xviii., pp. 217-26; *cf.* Voss, *op. cit.*, pp. 71-2). An example of the results of critical investigation into applied mathematics is the discovery—which has also obvious practical results in the avoidance of labour doomed to unfruitfulness—by Poincaré of limits of validity for certain of Laplace's formulæ.

[1] Mathematics is a wonderfully refined *symbolic* (for the importance of this character, see Voss, *op. cit.*, pp. 25-26) logic, the product of thousands of minds, and so adapted as to spare all waste of thought on unessentials.

THE IRON AND STEEL INSTITUTE.

THE fortieth annual general meeting of the Iron and Steel Institute was held at the Institution of Civil Engineers on May 13 and 14, under the chairmanship of Sir Hugh Bell, who retains the office of president for another year, and will be succeeded next May by his Grace the Duke of Devonshire. The report of the council for the past year shows that the affairs of the institute are in a prosperous condition. Five Carnegie research scholarships had been awarded, and Mr. Carnegie had presented 11,000 dollars, the income of which would assist in meeting clerical expenses and those incurred in issuing special memoirs.

The proceedings on May 13 opened with three papers, taken together for discussion, dealing with corrosion and protection of iron and steel. The paper by Mr. W. H. Walker, of Boston, U.S.A., contains the fundamental conceptions involved in the modern electrolytic theory of the corrosion of iron, develops this theory from the facts now known, and shows that the older carbonic-acid theory can be, and is, included therein, and points out some of the practical applications of this theory to the problem of corrosion. Mr. Allerton S. Cushman, of the United States Department of Agriculture, contributed a paper on the preservation of iron and steel. The author favours the view of corrosion as an electrochemical phenomenon, and deals with the questions of the production of a metal highly resistant to corrosion, of protective coatings, and of the passive condition which iron is capable of assuming. It seems to be a fact that carefully made open-hearth metal, in which the ordinary impurities are cut down to mere traces, and in which the heat treatment has been carefully controlled, is much more resistant to corrosion than the ordinary types of metal with a comparatively high percentage of impurities. The preservation of iron and steel by application of other metals to the surface, and of paint and other coatings, is fully discussed, and certain experiments having the object of determining their relative values under ordinary weathering conditions, which are now being carried out in America, are described and illustrated with photographs. Mr. J. Cruickshank Smith, of London, contributed a paper on physical tests for protective coatings for iron and steel. Tests are described for examining the following points:—that the proper proportion of pigment and vehicle has been obtained with the minimum of free oil space in the dry film; the smallness and uniformity of size of the pigmentary particles; the possession of the property of minimum tendency of the pigment and vehicle to separate; the determination of the thickness and uniformity of the film and its strength and elasticity; the permeability and hardness of the film.

An important paper on the solubility of steel in sulphuric acid was contributed by Messrs. E. Heyn and O. Bauer, of Gross-Lichterfelde. This paper contains 120 pages of matter, together with plates, and can only be briefly noticed here. The authors' researches show that the transition from the martensite of hardened steel to pearlite of annealed steel is not continuous through the intermediate stage of tempering as has been hitherto supposed. There is an intermediate metastable form to which the authors have given the name of "osmondite," in honour of Osmond. The fact is shown by the curve of solubility in dilute sulphuric acid attaining a sharply defined maximum at 400° C. The researches dealt with the influences of the quenching and tempering of steel on its solubility, of quenching and re-heating soft mild steel, and of the quenching temperature; the influence of cold working and annealing on the solubility of mild steel, and of the